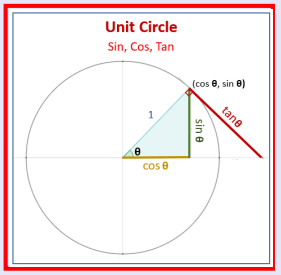
Math 241

Winter 2023

Lecture 10



Feb 19-8:47 AM

x	0° / 0	30° / $\pi/6$	45° / $\pi/4$	60° / $\pi/3$	90° / $\pi/2$
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

$\tan x = \frac{\sin x}{\cos x}$ at $x = 90^\circ$

$\tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0}$

Use Your calc To find

$\tan 80^\circ \approx 5.671$

$\tan 89^\circ \approx 57.290$

$\tan 85^\circ \approx 11.430$

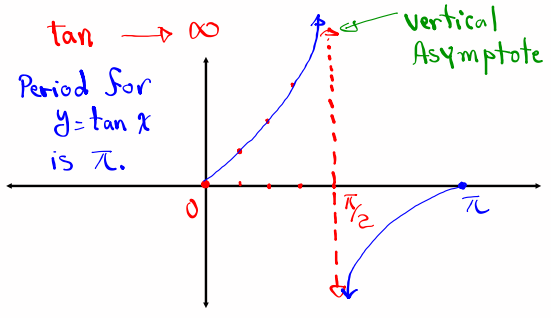
$\tan 89.999^\circ \approx 57295.780$

As we get close to 90° from left Side

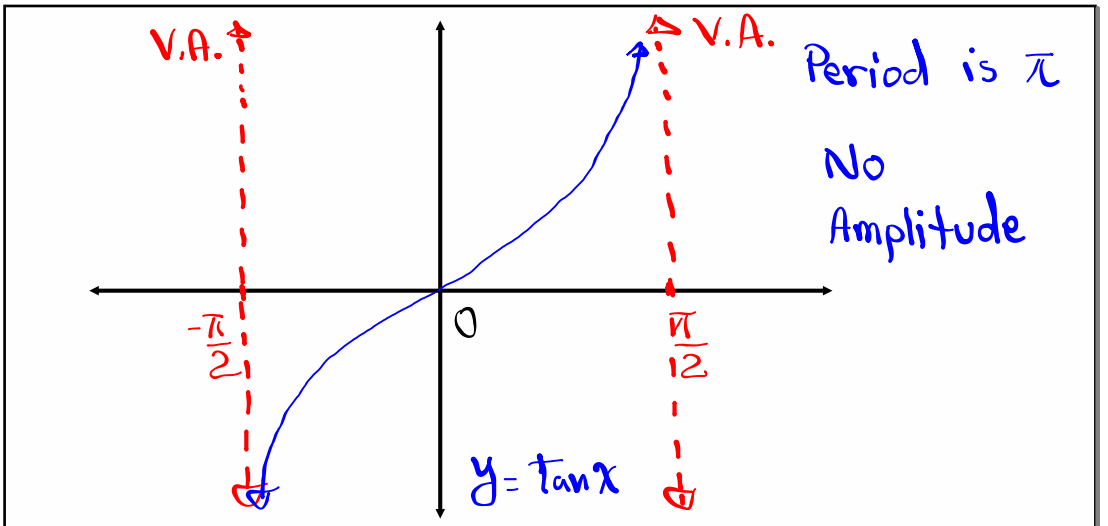
$\tan \to \infty$

Vertical Asymptote

Period for $y = \tan x$ is π.



Jan 19-7:02 AM



Jan 19-7:12 AM

Graph $y = \tan\left(\frac{1}{2}x\right)$

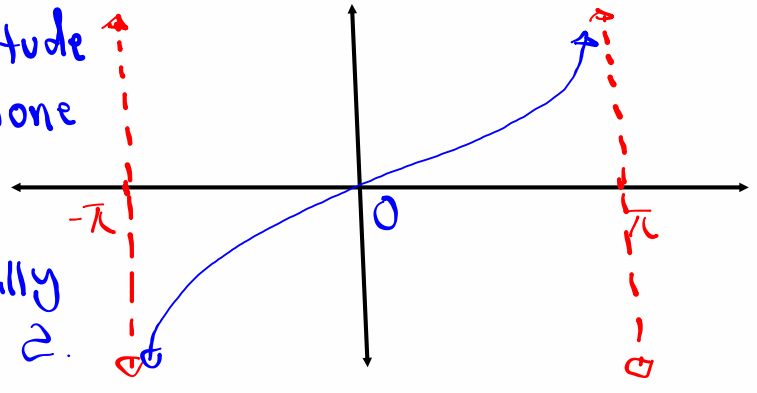
$-\frac{\pi}{2} < \frac{1}{2}x < \frac{\pi}{2}$

Multiply by 2

$-\pi < x < \pi$

x	$-\pi$	0	π
$\frac{1}{2}x$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
$\tan\frac{1}{2}x$	und. V.A.	0	und. V.A.

No Amplitude
shift None
Stretch horizontally by factor 2.



Jan 19-7:14 AM

Graph $y = \tan\left(x - \frac{\pi}{2}\right)$

$-\frac{\pi}{2} < x - \frac{\pi}{2} < \frac{\pi}{2}$

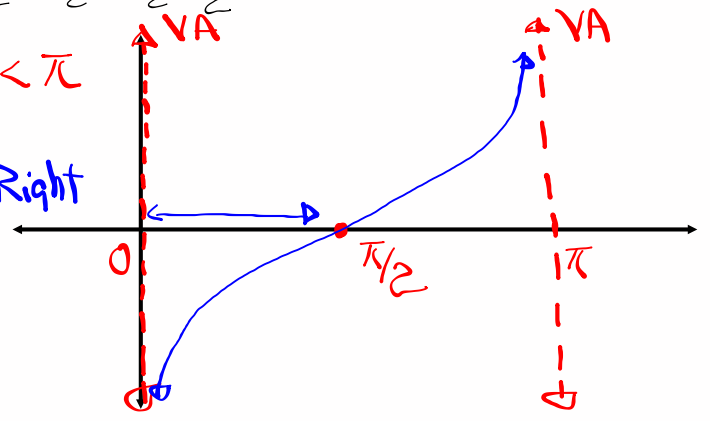
Add $\frac{\pi}{2}$

$-\frac{\pi}{2} + \frac{\pi}{2} < x - \frac{\pi}{2} + \frac{\pi}{2} < \frac{\pi}{2} + \frac{\pi}{2}$

$0 < x < \pi$

x	0	$\pi/2$	π
$x - \frac{\pi}{2}$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
$\tan(x - \frac{\pi}{2})$	und. V.A.	0	und. V.A.

phase shift horizontal $\pi/2$ Right



Jan 19-7:21 AM

Graph $y = \tan\left(2x + \frac{\pi}{2}\right)$

$-\frac{\pi}{2} < 2x + \frac{\pi}{2} < \frac{\pi}{2}$

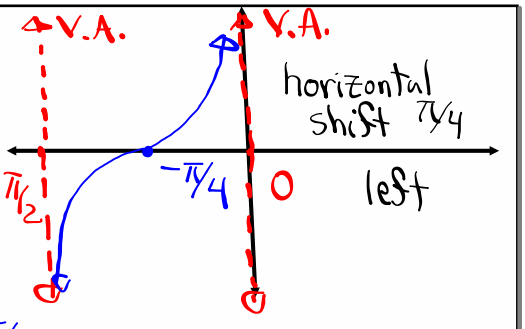
Subtract $\frac{\pi}{2}$

$-\pi/2 - \frac{\pi}{2} < 2x + \pi/2 - \pi/2 < \pi/2 - \pi/2$

$-\pi < 2x < 0$

Divide by 2

$-\frac{\pi}{2} < x < 0$



x	$-\pi/2$	$-\pi/4$	0
$2x + \pi/2$	$-\pi/2$	0	$\pi/2$
$\tan(2x + \pi/2)$	und. V.A.	0	und. V.A.

Jan 19-7:27 AM

Graph $y = \tan\left(\frac{\pi}{4}x\right)$

$-\frac{\pi}{2} < \frac{\pi}{4}x < \frac{\pi}{2}$

multiply by 4

$-2\pi < \pi x < 2\pi$

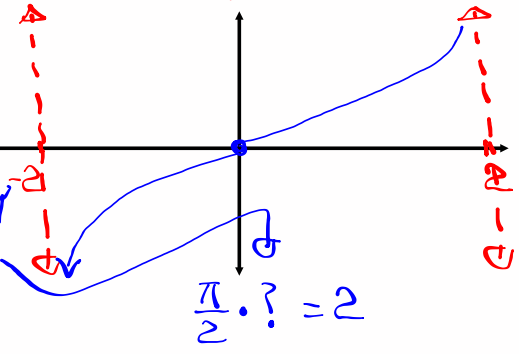
Divide by π

$-2 < x < 2$

No shifting

Stretch horizontally by a factor of $\frac{4}{\pi}$

x	-2	0	2
$\frac{\pi}{4}x$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
$\tan\left(\frac{\pi}{4}x\right)$	und. V.A.	0	und. V.A.



$\frac{\pi}{2} \cdot ? = 2$

Jan 19-7:37 AM

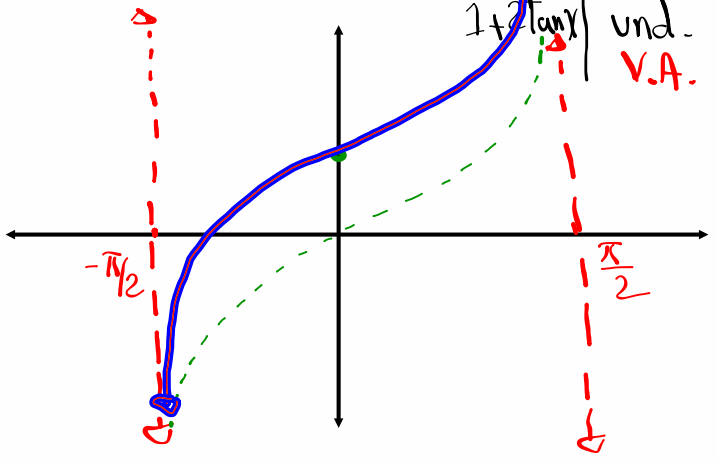
Graph $y = 1 + 2\tan x$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

Vertical Shift 1 up

x	$-\pi/2$	0	$\pi/2$
$\tan x$	und.	0	und.
$2\tan x$	und.	0	und.
$1+2\tan x$	und. V.A.	1	und. V.A.

VA VA



Jan 19-7:46 AM

Graph $y = -2 - \tan\left(x - \frac{\pi}{2}\right)$ — Vertical shift 2 down

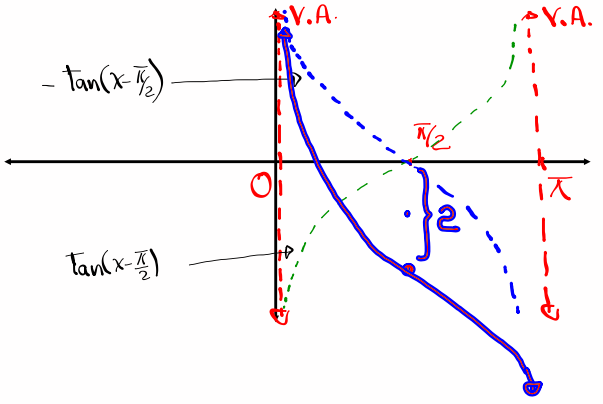
x	0	VA	$\pi/2$	VA	π
$x - \frac{\pi}{2}$	$-\pi/2$		0		$\frac{\pi}{2}$
$\tan(x - \frac{\pi}{2})$	und.		0		und.
$-2 - \tan(x - \frac{\pi}{2})$	und		-2		und.

$-\frac{\pi}{2} < x - \frac{\pi}{2} < \frac{\pi}{2}$

$\vdots$

$0 < x < \pi$

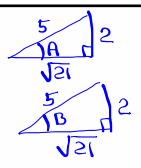
Cause Reflection by x-axis



Jan 19-7:52 AM

$\sin A = \frac{-2}{5}$ A in QIII

$\cos B = \frac{\sqrt{21}}{5}$ B in QIV



Find

1) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$= \frac{-2}{5} \cdot \frac{\sqrt{21}}{5} + \frac{-\sqrt{21}}{5} \cdot \frac{-2}{5} = \frac{-2\sqrt{21}}{25} + \frac{2\sqrt{21}}{25} = 0$

2) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= \frac{-\sqrt{21}}{5} \cdot \frac{\sqrt{21}}{5} + \frac{-2}{5} \cdot \frac{-2}{25} = \frac{-21}{25} + \frac{4}{25} = \frac{-17}{25}$

3) $\tan 2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{2 \cdot \frac{2}{\sqrt{21}}}{1 - \left(\frac{2}{\sqrt{21}}\right)^2}$

$= \frac{\frac{4}{\sqrt{21}}}{1 - \frac{4}{21}}$

LCD $= 21\sqrt{21}$

$= \frac{21\sqrt{21} \cdot \frac{4}{\sqrt{21}}}{21\sqrt{21} \cdot 1 - 21\sqrt{21} \cdot \frac{4}{21}}$

$= \frac{84}{21\sqrt{21} - 4\sqrt{21}} = \frac{84 \cdot \sqrt{21}}{17\sqrt{21} \cdot \sqrt{21}}$

$= \frac{84\sqrt{21}}{17 \cdot 21} = \frac{4\sqrt{21}}{17}$

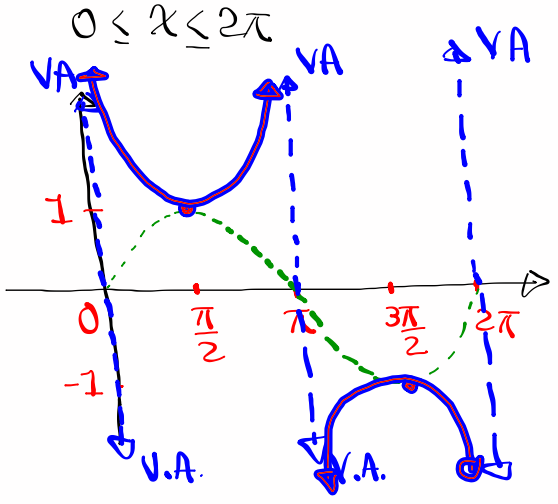
Jan 19-8:02 AM

Graph $y = \csc x$

1) Graph $y = \sin x$

2) Do reciprocal

3) V.A. at where $\sin x$ crosses x-axis.



Jan 19-8:34 AM

Graph $y = \csc\left(\frac{\pi}{2}x\right)$

1) Graph $y = \sin \frac{\pi}{2}x$

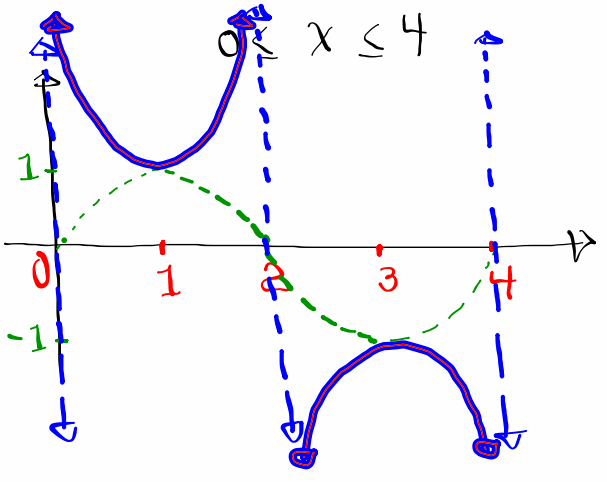
2) Draw V.A.

3) Draw reciprocals

$0 \le \frac{\pi}{2}x \le 2\pi$

$0 \le \pi x \le 4\pi$

$0 \le x \le 4$



Jan 19-8:38 AM

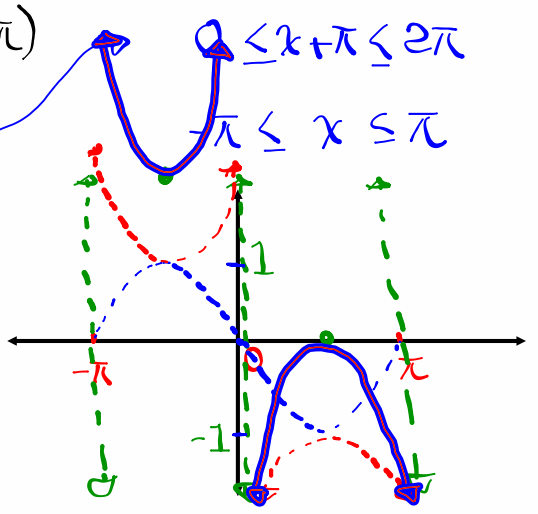
Graph $y = 1 + \csc(x+\pi)$

1) Graph $y = \sin(x+\pi)$

2) Draw V.A.

3) Graph the reciprocal

4) Shift up 1



Jan 19-8:42 AM

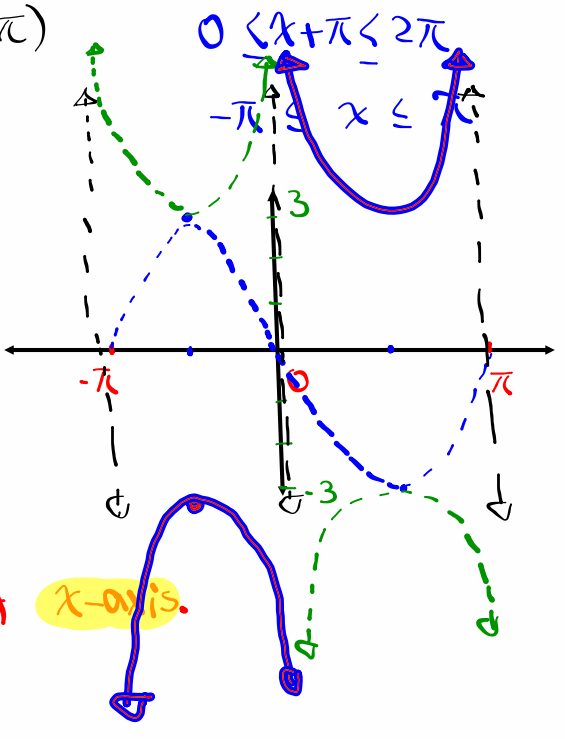
Graph $y = -3\csc(x+\pi)$

1) Graph $y = 3\sin(x+\pi)$

2) Draw V.A.

3) Draw reciprocal

4) take care of reflection about x-axis.



Jan 19-8:49 AM

Graph $y = -1 + \frac{1}{2}\csc(\pi x - \pi)$

1) Draw $y = \frac{1}{2}\sin(\pi x - \pi)$

2) Draw V.A.

3) Draw reciprocals

4) shift down 1.

$0 \le \pi x - \pi \le 2\pi$

Add π

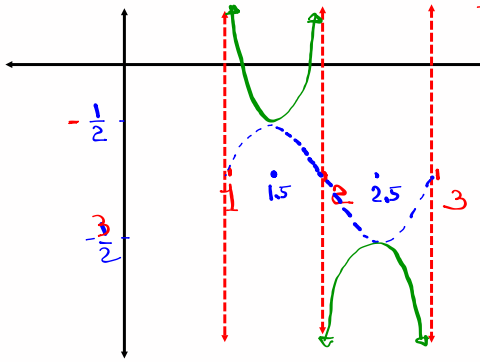
$\pi \le \pi x \le 3\pi$

Divide by π

$1 \le x \le 3$

$\frac{1}{2} - 1 = -\frac{1}{2}$

$-\frac{1}{2} - 1 = -\frac{3}{2}$



Jan 19-8:56 AM

Graph $y = \sec(x - \pi)$

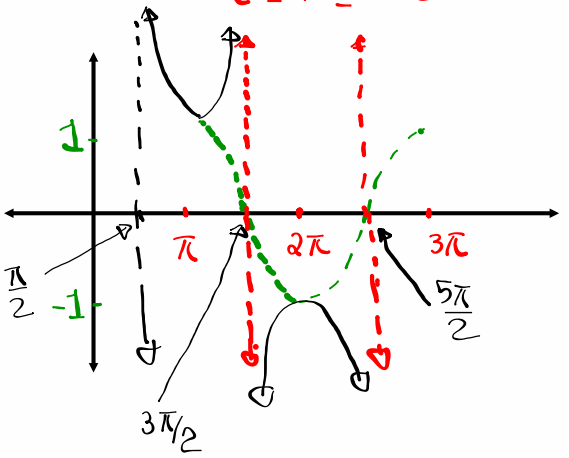
1) Graph $y = \cos(x - \pi)$

2) Draw V.A.

3) Draw reciprocal

$0 \le x - \pi \le 2\pi$

$\pi \le x \le 3\pi$



Jan 19-9:05 AM

Graph $y = -2\sec\left(\frac{\pi}{2}x\right)$

1) Graph $y = 2\cos\left(\frac{\pi}{2}x\right)$

$0 \le \frac{\pi}{2}x \le 2\pi$

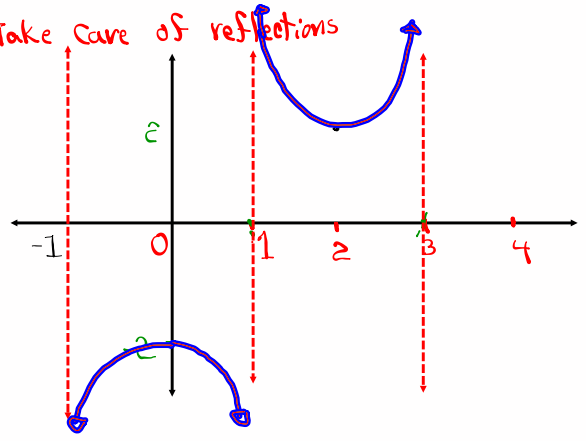
$0 \le \pi x \le 4\pi$

$0 \le x \le 4$

2) Draw V.A.

3) Draw reciprocals

4) Take care of reflections



Jan 19-9:18 AM

Graph $y = 3 + \sec(-x + \pi)$

Recall $\sin(-x) = -\sin x$ $\quad \csc(-x) = -\csc x$

$\cos(-x) = \cos x$ $\quad \sec(-x) = \sec x$

$\tan(-x) = -\tan x$ $\quad \cot(-x) = -\cot x$

$\sec(-x + \pi) = \sec[-(x - \pi)] = \sec(x - \pi)$

My Graph $y = 3 + \sec(x - \pi)$

1) Graph $y = \cos(x - \pi)$

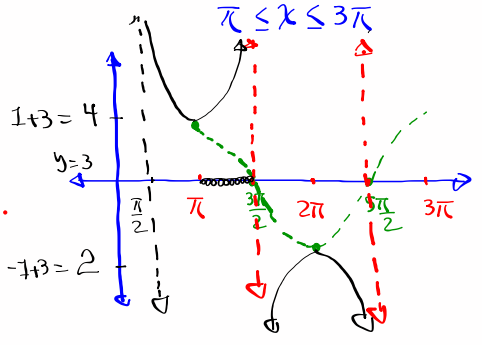
$0 \le x - \pi \le 2\pi$

$\pi \le x \le 3\pi$

2) Draw V.A.

3) Do reciprocals

4) shift 3 up.



Jan 19-9:31 AM

Graph $y = 3 + \sec(-x + \pi)$

1) Graph $y = \cos(-x + \pi)$

2) Draw V.A.

3) Draw reciprocal

4) Vertical Shift 3 up.

$0 \le -x + \pi \le 2\pi$

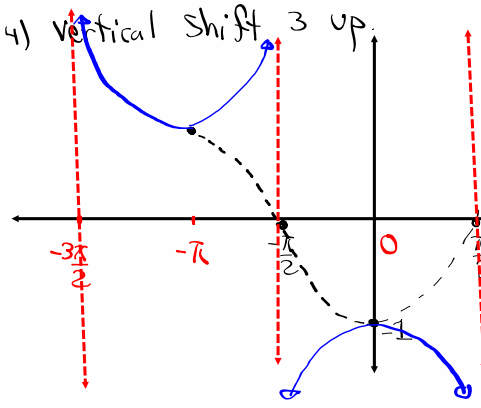
$-\pi \le -x \le \pi$

Multiply by -1

$\pi \ge x \ge -\pi$

Same as

$-\pi \le x \le \pi$



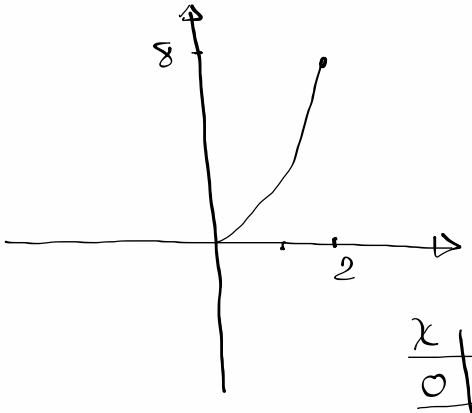
Finish with vertical shift 3 up.

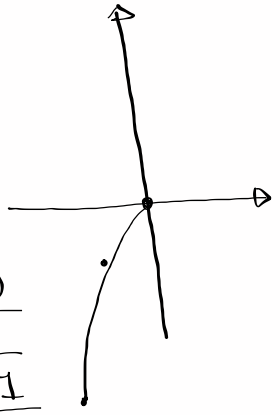
Jan 19-9:46 AM

$y = x^3$

x	y
0	0
1	1
2	8

$0 \le x \le 2$



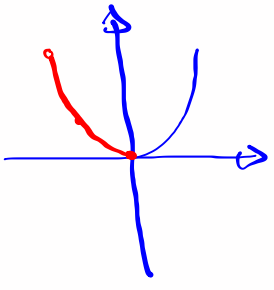


x	y
0	0
-1	-1
-2	-8

$y = x^2$

x	y
0	0
1	1
2	4

$0 \le x \le 2$



x	y
0	0
-1	1
-2	4

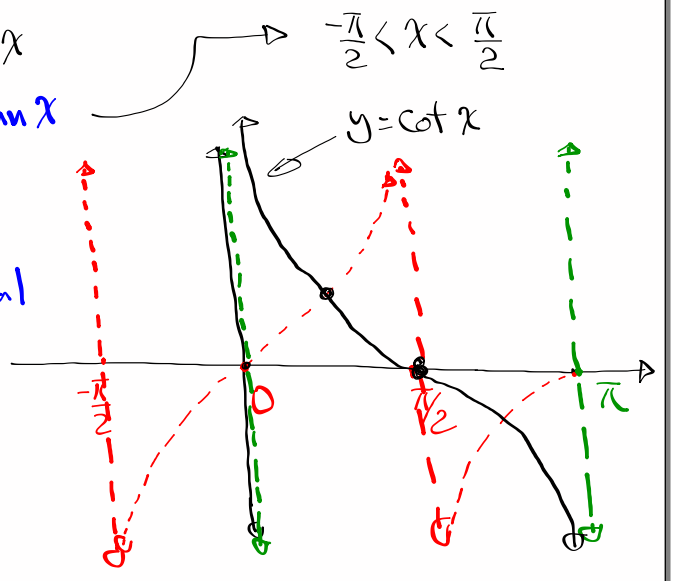
Jan 19-9:56 AM

Graph $y = \cot x$

1) Graph $y = \tan x$

2) Draw V.A.

3) Do reciprocal



for $\cot x$

$0 < \quad < \pi$

V.A. at left & right

Cross x-axis at middle point.

Jan 19-10:19 AM

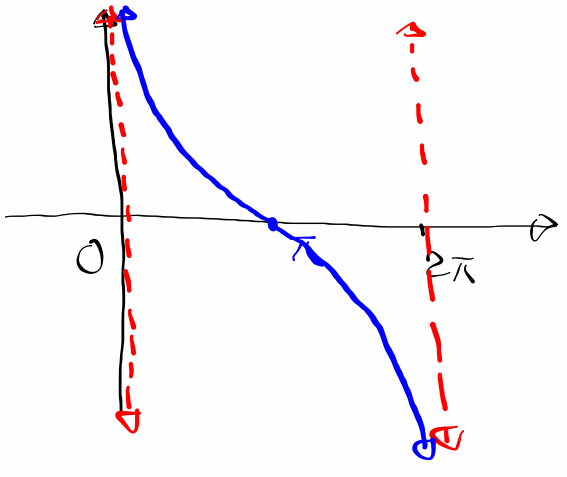
Graph $y = \cot\left(\frac{1}{2}x\right)$

$0 < \frac{1}{2}x < \pi$

$0 < x < 2\pi$

V.A. at 0, and 2π

Cross x-axis at π.



Jan 19-10:25 AM

Graph $y = -\cot\left(\frac{1}{4}x + \frac{\pi}{2}\right)$

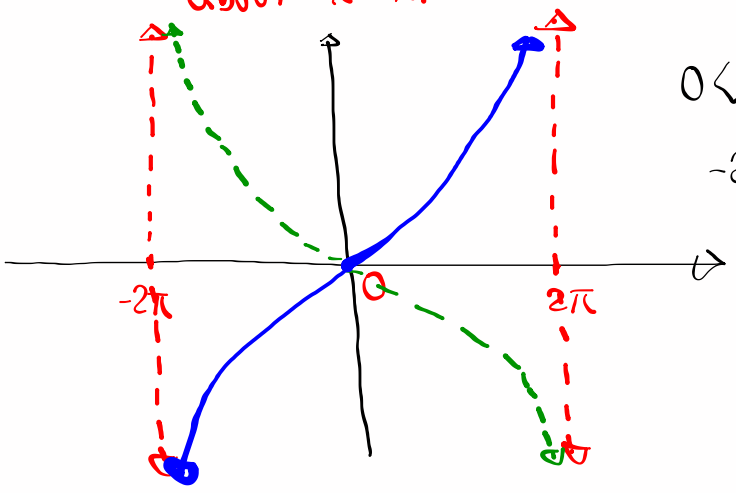
Reflection about x-axis

$0 < \frac{1}{4}x + \frac{\pi}{2} < \pi$

LCD = 4

$0 < x + 2\pi < 4\pi$

$-2\pi < x < 2\pi$



Jan 19-10:28 AM

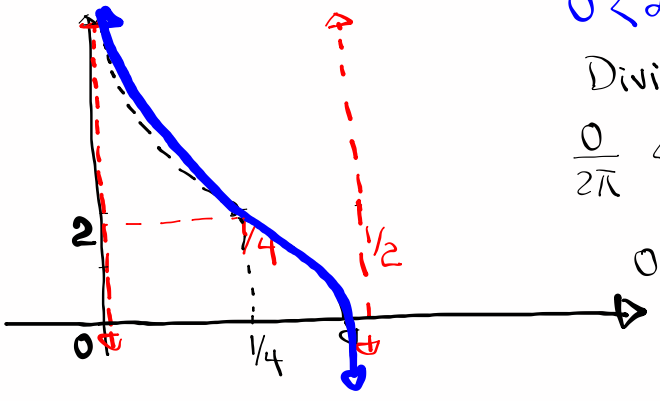
Graph $y = 2 + \cot(2\pi x)$

$0 < 2\pi x < \pi$

Divide by 2π

$\frac{0}{2\pi} < x < \frac{\pi}{2\pi}$

$0 < x < \frac{1}{2}$



Let's trace this graph.

If $x = \frac{1}{4}$, $y = 2 + \cot\left(2\pi \cdot \frac{1}{4}\right) = 2 + \cot \pi/2$

$= 2 + 0$

$= \boxed{2}$

$\cot \pi/2 = \frac{\cos \pi/2}{\sin \pi/2} = \frac{0}{1} = 0$

Jan 19-10:32 AM

Graph $y = 4 \sin x \cos x$

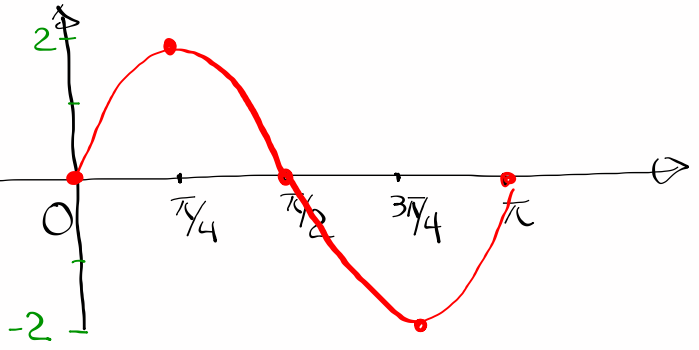
$y = 2 \cdot 2 \sin x \cos x$

$y = 2 \sin 2x$

Amplitude

$0 \le 2x \le 2\pi$

$0 \le x \le \pi$



Jan 19-10:39 AM

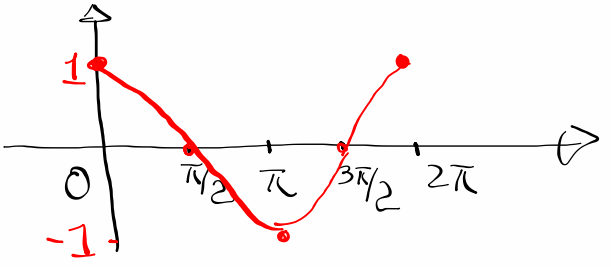
Graph $y = \cos^2 \frac{1}{2}x - \sin^2 \frac{1}{2}x$

Recall

$\cos 2x = \cos^2 x - \sin^2 x$

$y = \cos 2\left(\frac{1}{2}x\right)$

$y = \cos x$



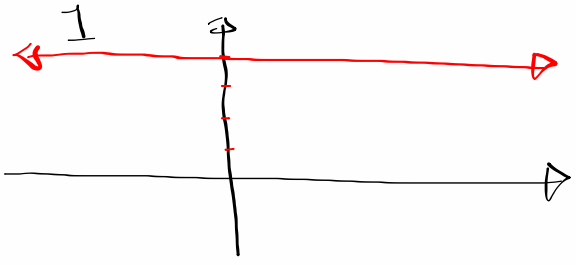
Jan 19-10:43 AM

Graph $y = 4\sin^2 x + 4\cos^2 x$

$= 4(\sin^2 x + \cos^2 x)$

$= 4(1)$

$y = 4$



Jan 19-10:47 AM

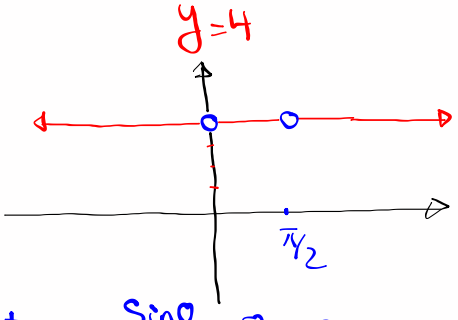
Graph $y = 4\tan x \cot x$

$\tan x \cot x = 1$

$5 \cdot \frac{1}{5} = 1$

$\frac{2}{3} \cdot \frac{3}{2} = 1$

$0 \cdot \frac{1}{0} = 1$



If $x = 0$

$y = 4\tan 0 \cdot \cot 0$

$= 4 \cdot 0 \cdot$

If $x = \frac{\pi}{2}$

$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

$\cot 0 = \frac{1}{\tan 0} = \frac{1}{0}$ undefined.

$\tan \frac{\pi}{2} = \frac{\sin \pi/2}{\cos \pi/2} = \frac{1}{0}$ undefined.

Jan 19-10:49 AM

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$ | $\cos 2x = 1 - 2\sin^2 x$

$1 + \cos 2x = 2\cos^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\frac{1 + \cos 2x}{2} = \cos^2 x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos x = \pm\sqrt{\frac{1 + \cos 2x}{2}}$

$\sin x = \pm\sqrt{\frac{1 - \cos 2x}{2}}$

Half-Angle Formula

$\tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$

$\tan\frac{x}{2} = \frac{\sin x}{1 + \cos x}$

$\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x}$

Jan 19-10:58 AM

Find exact value of

$\tan 22.5^\circ = \tan\frac{45^\circ}{2} = \frac{\sin 45^\circ}{1 + \cos 45^\circ}$

$\tan\frac{x}{2} = \frac{\sin x}{1 + \cos x}$

$= \frac{\frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2 + \sqrt{2}}$

$= \frac{\sqrt{2}(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})} = \frac{2\sqrt{2} - \sqrt{4}}{4 - 2\sqrt{2} + 2\sqrt{2} - \sqrt{4}} = \frac{2\sqrt{2} - 2}{4 - 2}$

$= \frac{2\sqrt{2} - 2}{2}$

$= \frac{2(\sqrt{2} - 1)}{2}$

$\tan 22.5^\circ = \sqrt{2} - 1$

Jan 19-11:05 AM

Find exact Value for $\sin 15^\circ$.

$15^\circ = 45^\circ - 30^\circ$, $15^\circ = \frac{30^\circ}{2}$

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$ $\quad \sin(A-B) = \sin A\cos B - \cos A \sin B$

$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

QI

$\sin 15^\circ = \sin\frac{30^\circ}{2} = \pm\sqrt{\frac{1-\cos 30^\circ}{2}}$ $\quad \approx \boxed{.259}$

$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\approx \boxed{.259}$ ✓

Jan 19-11:09 AM

Show $\frac{\sqrt{6}-\sqrt{2}}{4} = \frac{\sqrt{2-\sqrt{3}}}{2}$

$2(\sqrt{6}-\sqrt{2}) \stackrel{?}{=} 4\sqrt{2-\sqrt{3}}$

$2(\sqrt{6}-\sqrt{2}) \stackrel{?}{=} \sqrt{16(2-\sqrt{3})}$

$\left[2(\sqrt{6}-\sqrt{2})\right]^2 \stackrel{?}{=} \left[\sqrt{16(2-\sqrt{3})}\right]^2$ ✓

$4 \neq -4$

$4^2 = (-4)^2$

$16 = 16$

$4(\sqrt{6}-\sqrt{2})(\sqrt{6}-\sqrt{2}) \stackrel{?}{=} 16(2-\sqrt{3})$

$4(6-\sqrt{12}-\sqrt{12}+2) \stackrel{?}{=} 16(2-\sqrt{3})$

$4(8-2\sqrt{12}) \stackrel{?}{=} 16(2-\sqrt{3})$

$4(8-2\sqrt{4}\sqrt{3}) \stackrel{?}{=} 16(2-\sqrt{3})$

$4(8-4\sqrt{3}) \stackrel{?}{=} 16(2-\sqrt{3})$

$4\cdot 4(2-\sqrt{3}) \stackrel{?}{=} 16(2-\sqrt{3})$

Jan 19-11:18 AM

$\sqrt{x^2} = x$ if $x \geq 0$

try to work on one side to get to other side to show

$$\frac{\sqrt{6}-\sqrt{2}}{4} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

Jan 19-11:29 AM